SUGGESTED SOLUTION

SYJC 2020

SUBJECT- MATHS AND STATS

Test Code - SYJ 6088

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ANSWER: 1

(A) **(02)**

$$A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$$

Here, $a_{11} = 2, M_{11} = 5$

$\therefore\ A_{11} = (-1)^{1+1}(5) = 5$

$a_{12} = -3, M_{12} = 3$

$\therefore\ A_{12} = (-1)^{1+2}(3) = -3$

$a_{21} = 3, M_{21} = -3$

$\therefore\ A_{21} = (-1)^{2+1}(-3) = 3$

$a_{22} = 5, M_{22} = 2$

$\therefore\ A_{22} = (-1)^{2+2} = 2.$

$\therefore$ the cofactor matrix = $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}$

$\therefore$ adj A = $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

(B) **(2)**

$f(x) = \dfrac{3x^2 + 4x + 9}{x^2 - 6x + 10}$ being a rational function is discontinuous at those points where its denominator $x^2 - 6x + 10 = 0$. i.e., when $x = \dfrac{6 \pm \sqrt{36-40}}{2} = \dfrac{6 \pm \sqrt{-4}}{2}$ which are not real numbers. Hence, *f* is continuous for all $x \in R$.

(C) **(2)**

$f(0) = k$ (Given) .... (1)

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(e^x - 1)\sin x}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{e^x - 1}{x}\right) . \frac{\sin x}{x}$$

$$= \lim_{x \to 0} \frac{e^x - 1}{x} \text{ x } \lim_{x \to 0} \frac{\sin x}{x}$$

= 1 x 1 = 1 .... (2)

Since f is continuous at $x = 0$,

$$f(0) = \lim_{x \to 0} f(x)$$

$\therefore$ $k = 1$. ....[By (1) and (2)]

(D) **(02)**

$$\left\{3\begin{bmatrix}1 & 2 & 0\\0 & -1 & 3\end{bmatrix}-\begin{bmatrix}1 & 5 & -2\\-3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$= \left\{\begin{pmatrix}3 & 6 & 0\\0 & -3 & 9\end{pmatrix}-\begin{bmatrix}1 & 5 & -2\\-3 & -4 & 4\end{bmatrix}\right\}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1 & 2\\3 & 1 & 5\end{bmatrix}\begin{bmatrix}1\\2\\1\end{bmatrix}$$

$$= \begin{bmatrix}2+2+2\\3+2+5\end{bmatrix}=\begin{bmatrix}6\\10\end{bmatrix}$$

(E) **(02)**

$$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\therefore \begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\therefore \begin{bmatrix}2+y & 6+0\\0+1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\therefore \begin{bmatrix}2+y & 6\\1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

By equality of matrices, we get,

$2+y=5$ **and** $2x+2=8$

$\therefore$ $y=3$ **and** $2x=6$ $\quad\quad$ $\therefore x=3$

$\therefore$ $x=3$ **and** $y=3$.

(F) **(02)**

$f(x)=x^2-x+9,$ for $x \le 3$

$\therefore$ $f(3)=9-3+9=15$ .... (1)

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3} (x^2 - x + 9)$

= 9 -3+ 9 =15 .... (2)

Also, $f(x) = 4x + 3$, for $x > 3$

$\therefore \lim_{x \to 3^+} f(x) = \lim_{x \to 3} (4x + 3)$

= 4 (3) +3 =15 ....(3)

From (1), (2) and (3),

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

$\therefore$ *f* is continuous at *x* = 3.

ANSWER : 2

(A) **(03)**

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 4 - 3 = 1 \neq 0$$

$\therefore$ A^{-2} exists.

We write $AA^{-1}=I$

$$\therefore \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By $R_1 \leftrightarrow R_2$, $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

By $R_2 - 2R_1$, $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$

By $(-1)R_2$, $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$

By $R_2 - 2R_2$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

(B) **(3)**

Given *f* is continuous at *x* = 1.

$\therefore \quad f(1) = \lim_{x \to 1} f(x)$

$$= \lim_{x \to 1} \frac{1+\cos \pi x}{(1-x)^2}$$

Put $(1-x) = t$ $\quad \therefore x = 1-t$

as $x \to 1, t \to 0$

$$\therefore f(1) = \lim_{t \to 0} \frac{1+\cos[\pi(1-t)]}{t^2}$$

$$= \lim_{t \to .} \frac{1+\cos(\pi - \pi t)}{t^2}$$

$$= \lim_{t \to 0} \frac{1-\cos \pi t}{t^2}$$

$$= \lim_{t \to 0} \frac{1-\cos \pi t}{t^2} \text{ x } \frac{1+\cos \pi t}{1+\cos \pi t}$$

$$\left(\text{as } t \to 0; (1+\cos t) \neq 0\right)$$

$$= \lim_{t \to 0} \frac{1-\cos^2 \pi t}{(1+\cos \pi t)}$$

$$= \lim_{t \to 0} \frac{\sin^2 \pi t}{(1+\cos \pi t)}$$

$$= \lim_{t \to 0} \left(\frac{\sin \pi t}{\pi t}\right)^2 \text{ x } \frac{\pi^2}{(1+\cos \pi t)}$$

$$= 1 \text{ x } \frac{\pi^2}{2}$$

$$= \frac{\pi^2}{2}$$

$$\therefore f(1) = \frac{\pi^2}{2}$$

(C) **(3)**

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{15^x - 3^x - 5^x + 1}{x \tan x}$$

$$= \lim_{x \to 0} \frac{3^x . 5^x - 3^x - 5^x + 1}{x \tan x}$$

$$= \lim_{x \to 0} \frac{3^x (5^x - 1) - 1(5^x - 1)}{x \tan x}$$

$$= \lim_{x \to 0} \frac{(5^x - 1).(3^x - 1)}{x \tan x}$$

$$= \lim_{x \to 0} \frac{\left(\frac{5^x - 1}{x}\right) . \left(\frac{3^x - 1}{x}\right)}{\left(\frac{\tan x}{x}\right)} \quad ...[x \to 0, x \neq 0]$$

$$= \frac{\left(\lim_{x \to 0} \frac{5^x - 1}{x}\right) . \left(\lim_{x \to 0} \frac{3^x - 1}{x}\right)}{\left(\lim_{x \to 0} \frac{\tan x}{x}\right)}$$

$$= \frac{(\log 5).(\log 3)}{1} \; ... \left[\because \lim_{x \to 0} \frac{a^x - 1}{x} = \log a\right]$$

$$\therefore \lim_{x \to 0} f(x) = (\log 5).(\log 3)$$

Now, f is continuous at $x = 0$

$$\therefore \; f(0) = \lim_{x \to 0} f(x)$$

$$\therefore \; f(0) = (\log 5).(\log 3).$$

(D) **(03)**

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

By $2R_2$, $\begin{bmatrix} 2 & 1 \\ 6 & 10 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$

By $R_2 - 3R_1$

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2x + y \\ 0 + 7y \end{bmatrix} = \begin{bmatrix} 5 \\ -21 \end{bmatrix}$$

By equality of matrices,

$2x + y = 5$ .... (1)

$7y = -21$ .... (2)

From (2), $y = -3$

Substituting $y = -3$ in (1), we get,

$2x - 3 = 5$

$\therefore \; 2x = 8 \qquad \therefore x = 4$

Hence, $x = 4, y = -3$ is the required solution.

ANSWER : 3

(A) **(4)**

$$\lim_{x \to 1^{-1}} f(x) = \lim_{x \to 1} \frac{x^2 + \sqrt{x} - 2}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\left(x^2 - 1\right) + \left(\sqrt{x} - 1\right)}{x^2 - 1}$$

$$= \lim_{x \to 1} \frac{\left(\frac{x^2 - 1}{x - 1}\right) + \left(\frac{x^{\frac{1}{2}} - 1}{x - 1}\right)}{\left(\frac{x^2 - 1}{x - 1}\right)}$$

.... $\left[x \to 1, \neq 1 \qquad \therefore x - 1 \neq 0\right]$

$$= \frac{\lim_{x \to 1}\left(\frac{x^2 - 1}{x - 1}\right) + \lim_{x \to 1}\left(\frac{x^{\frac{1}{2}} - 1}{x - 1}\right)}{\lim_{x \to 1}\left(\frac{x^2 - 1}{x - 1}\right)}$$

$$= \frac{2.1^{2-1} + \frac{1}{2}.1^{\frac{1}{2}-1}}{2.1^{2-1}} \ldots \left[\because \lim_{x \to 1} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$= \frac{2 + \frac{1}{2}}{2} = \frac{\frac{3}{2}}{2} = \frac{3}{4}$$ .... (1)

$$\lim_{x \to 1^{+}} f(x) = \lim_{x \to 1} \frac{\sqrt{x+3} - 2}{\sqrt{2-x} - 1}$$

$$= \lim_{x \to 1} \frac{\sqrt{x+3} - 2}{\sqrt{2-x-1}} \text{ x } \frac{\sqrt{2-x} + 1}{\sqrt{2-x} + 1} \text{ x } \frac{\sqrt{x+3} + 2}{\sqrt{x+3} + 2}$$

$$= \lim_{x \to 1} \frac{(x+3-4)\left(\sqrt{2-x} + 1\right)}{(2-x-1)\left(\sqrt{x+3} + 2\right)}$$

$$= \lim_{x \to 1} \frac{(x-1)\left(\sqrt{2-x} + 1\right)}{-(x-1)\left(\sqrt{x+3} + 2\right)}$$

$$= \lim_{x \to 1} \frac{\sqrt{2-x} + 1}{\left(\sqrt{x+3} + 2\right)}$$

.... $\left[x \to 1, x \neq 1 \qquad \therefore x - 1 \neq 0\right]$

$$= \frac{\lim_{x \to 1} \left(\sqrt{2-x}+1\right)}{\lim_{x \to 1} \left(\sqrt{x+3}+2\right)}$$

$$= \frac{\sqrt{2-1}+1}{-\left(\sqrt{1+3}+2\right)} = \frac{1+1}{-(2+2)} = -\frac{2}{4} = -\frac{1}{2}$$ .... (2)

From (1) and (2),

$$\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$$

$\therefore \lim_{x \to 1} f(x)$ does not exist

$\therefore f$ is discontinuous at $x = 1$.

This discontinuity cannot be removed.

(B) **(04)**

Let the three numbers be x, y and z.

$\therefore \; x+y+z=6.$

According to the given conditions,

$3z+y=11,$ i.e., $y+3z=11$

and $x+z=2y,$ i.e., $x-2y+z=0$

Hence, the system of the linear equations is

$x+y+z=6$

$y+3z=11$

$x-2y+z=0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

By $R_3 - R_1$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ -6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x+y+z \\ 0+y+3z \\ 0-3y+0 \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ -6 \end{bmatrix}$$

By equality of matrices,,

$x+y+z=6$ .... (1)

$y+3z=11$ .... (2)

$-3y=-6$ .... (3)

From (3), y= 2

Substituting y = 2 in (2), we get,

$2+3z=11$

$\therefore\ 3z=9 \qquad \therefore z=3$

Put y =2, z = 3 in (1), we get,

$x+2+3=6 \quad \therefore x=1$

$\therefore\ x=1 y=2, z=3$

Hence, the required numbers are 1,2 and 3.

(C) **(04)**

The given equations can be written in the matrix form as :

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

This is of the form AX= B, where

$$A=\begin{bmatrix} 1 & -1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and B=}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

Let us find A^{-1}.

$$|A|=\begin{vmatrix} 1 & -1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix}$$

$$=(-5-7)+1(-2-14)+1(2-10)$$

$$=-12-16-8=-36\neq 0$$

$\therefore$ A^{-1} exists.

We write AA^{-1} = 1

$$\therefore \begin{bmatrix} 1 & -1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}A^{-1}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

By R_2-2R_1 and R_3-2R_1,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 7 & 5 \\ 0 & 3 & -3 \end{bmatrix}A^{-1}=\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

by $R_2 - 2R_3$,

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 11 \\ 0 & 3 & -3 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ -2 & 0 & 1 \end{bmatrix}$$

By R_1+R_2 and $R_3 - 3R_2$,

$$\begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 11 \\ 0 & 0 & -36 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & -2 \\ -8 & -3 & 7 \end{bmatrix}$$

By $\left(\frac{1}{36}\right)$ R_3,

$$\begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 11 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 1 & -2 \\ \frac{8}{36} & \frac{3}{36} & \frac{7}{36} \end{bmatrix}$$

By R_1- $12R_3$andR_2 -$11R_3$,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{16}{36} & \frac{3}{36} & \frac{5}{36} \\ \frac{8}{36} & \frac{3}{36} & -\frac{7}{36} \end{bmatrix}$$

$$\therefore\ A \neq -1 = \frac{1}{36}\begin{bmatrix} 12 & 0 & 12 \\ -16 & 3 & 5 \\ 8 & 3 & -7 \end{bmatrix}$$

Now, premultiply AX= B by A^{-1}, we get,

$$A^{-1}(AX) = A^{-1}B$$

$$\therefore\ \left(A^{-1}A\right)X = A^{-1}B$$

$\therefore$ IX =A^{-1} B

$$\therefore X = \frac{1}{36}\begin{bmatrix} 12 & 0 & 12 \\ -16 & 3 & 5 \\ 8 & 3 & -7 \end{bmatrix}\begin{bmatrix} 9 \\ 52 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 108+0+0 \\ -144+156+0 \\ 72+156-0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{36}\begin{bmatrix} 108 \\ 12 \\ 228 \end{bmatrix} = \begin{bmatrix} 3 \\ \frac{1}{3} \\ \frac{19}{3} \end{bmatrix}$$

$\therefore$ by equality of the matrices,

$x = 3, y = \frac{1}{3}, z = \frac{19}{3}$ is the required solution.

(D) **(4)**

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} \frac{\tan x - \sin x}{\sin 3x - 3\sin x}$$

$$= \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{3\sin x - 4\sin^3 x - 3\sin x}$$

$$= \lim_{x \to 0} \frac{\sin x - \sin x \cos x}{\cos x\left(-4\sin^3 x\right)}$$

$$= \lim_{x \to 0} \frac{\sin x\left(1 - \cos x\right)}{-4\cos x.\sin^3 x}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{-4\cos x.\sin^2 x}$$

.. $\left[x \to 0, x \neq 0 \quad \therefore \sin x \neq 0\right]$

$$= -\frac{1}{4} \lim_{x \to 0} \frac{1 - \cos x}{\cos x\left(1 - \cos^2 x\right)}$$

$$= -\frac{1}{4} \lim_{x \to 0} \frac{1 - \cos x}{\cos x\left(1 - \cos x\right)\left(1 + \cos x\right)}$$

$$= -\frac{1}{4} \lim_{x \to 0} \frac{1}{\cos x\left(1 + \cos x\right)}$$

.... $\left[x \to 0, x \neq 0 \quad \therefore \cos x \neq \cos 0, \cos x \neq 1 \;\; \therefore 1 - \cos x \neq 0\right]$

$$= \frac{1}{4} \text{ x } \frac{1}{\lim_{x \to 0} \cos x \text{ x } \lim_{x \to 0}\left(1 + \cos x\right)}$$

$$= \frac{1}{4} \text{ x } \frac{1}{1\left(1+1\right)} = -\frac{1}{8}$$ ....(1)

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} \frac{3\sin^2 x}{3x^2} - 2\sin\left(x^2\right)$$

$= \dfrac{1}{3} \lim_{x \to 0} \left[\dfrac{\sin^2 x}{x^2} - \dfrac{2\sin\left(x^2\right)}{x^2} \right]$

$= \dfrac{1}{3} \lim_{x \to 0} \left[\left(\dfrac{\sin x}{x} \right)^2 - 2\dfrac{\sin\left(x^2\right)}{x^2} \right]$

$= \dfrac{1}{3} \left[\left(\lim_{x \to 0} \dfrac{\sin x}{x} \right)^2 - 2 \lim_{x \to 0} \dfrac{\sin\left(x^2\right)}{x^2} \right]$

$= \dfrac{1}{3} \left[(1)^2 - 2(1) \right]$

.... $\left[x \to 0, x^2 \to 0 \text{ and } \lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1 \right]$

$= \dfrac{1}{3}(1-2) = -\dfrac{1}{3}$ .... (2)

From (1) and (2),

$\lim_{x \to 0^-} \mathrm{f}(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} f(x)$ does not exist

$\therefore$ *f* is discontinuous at *x* =0.

Alternative Method :

$f(x) = \dfrac{3\sin^2 x - 2\sin\left(x^2\right)}{3x^2}, \text{ for } x \geq 0$

$f(x)$ is not defined at *x* = 0

$\therefore f(0)$ does not exist.

$\therefore$ *f* is discontinuous at *x* = 0.